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WEAPONS SYSTEM PRECISION

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ABSTRACT This article presents one type of method for studying precision--the Montecarlo method. It discusses in detail several keys to utilizing Montecarlo methods--the production of initial state sets, the introduction of error, the selection of statistical sets. Finally, it gives two examples of the use of Montecarlo methods.

KEY TERMS Precision analysis Ballistic missile Montecarlo method Error analysis

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I. OBJECTIVES

Antisatellite and antitactical ballistic missile weapons, as a group, all opt for the use of precision guidance to make terminal guidance target miss quantities very small--making it possible to directly hit targets. This then requires--with a certain terminal guidance control capability--that delivery vehicles have relatively high guidance precisions and targets have comparatively high measurement precision. As a result, there is a need to study guidance precision, target measurement precision, and the influences of terminal guidance control capabilities on systems, causing a rationalization of weapons system indices.

II. THE SETTING UP OF MODELS

1. Simple Introduction to Methods

There are many types of precision research methods. However, making use of Montecarlo methods is capable of simply and accurately obtaining the needed results. The draw back is that the amount of calculation is great. However, following along with the development of computers, this problem has already reached a very good solution. This article introduces how to make use of Montecarlo methods to carry out precision analysis. The schematic is shown by the use of Fig.1.

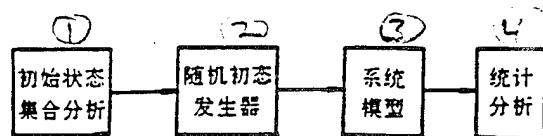


Fig.1 Montecarlo Schematic

Key: (1) Initial State Set Analysis (2) Random Initial State Generator (3) System Model (4) Statistical Analysis

This method is divided into four steps. First, analyze probability characteristics associated with initial state sets. Second, from initial state set probability characteristics, produce suitable random initial states. Third, take random initial states and substitute into system models to carry out simulation calculations. Fourth, with regard to the results of simulation calculations, carry out statistical analysis, obtaining needed precision indices.

2. Several Problems Associated with the Area of Relevant Probability Theory

(1) Initial State Set Analysis and Selection of Sample Numbers Initial states are talked about relative to system models. With respect to a system model, if one is given an initial state, in accordance with this model's calculations, it is possible to obtain values for a parameter in which there is interest or random processes. In accordance with the ideas of statistics, if it is possible to reasonably obtain the number of samples and the value of samples, it is then possible to make use of initial state sets associated with limited samples to accurately approximate initial state sets associated with unlimited samples. As a result, analysis of initial state sets is then very important.

Generally, when studying precision problems, the initial states are made up from error coefficients in error models. On the basis of knowledge about probability theory, it is possible to know that error coefficients obey normal distributions. Due to the fact that these error coefficients are mutually independent in this article, use is, therefore, made of mean values and variances. It is then possible to completely describe probability characteristics associated with initial states.

The selection of initial state sample numbers is very important. With sample numbers small, the credibility of statistical results is very low. With sample numbers large, it makes large amounts of calculations increase, but the

significance is not great. From statistical theory, it is possible to know that:

$$\bar{m}_z = \frac{1}{N} \sum_{i=1}^N X_i$$

$$\bar{\sigma}_z = \sqrt{\frac{1}{N} \sum_{i=1}^N (X_i - \bar{m}_z)^2}$$

In the equations, X_i is needed statistical random variables. N is the sample number. When random variable follow normal distributions, statistical quantities are then:

$$t = \frac{\bar{m} - m}{\bar{\sigma} / \sqrt{N}}$$

follows t distribution
associated with a degree
of freedom which is $N-1$

$$X^2 = \frac{(N-1) \bar{\sigma}^2}{\sigma^2}$$

follows X^2 distribution
associated with a degree
of freedom which is $N-1$

In the equations, σ and m are real variances and mean values associated with sets.

If one makes $1-\alpha$ be the degree of credibility associated with sample mean values and variances, then, the credibility zones associated with sample mean values and variances are:

mean value:

$$\text{均值: } \left[\bar{m} - t_{\alpha, N-1} \frac{\bar{\sigma}}{\sqrt{N}}, \bar{m} + t_{\alpha, N-1} \frac{\bar{\sigma}}{\sqrt{N}} \right]$$

In this

$$P \left\{ \left| \frac{\bar{m} - m}{\bar{\sigma} / \sqrt{N}} \right| \leq t_{\alpha, N-1} \right\} = 1 - \alpha$$

variance:

$$\text{方差: } \left[\bar{\sigma} \cdot \sqrt{\frac{N-1}{X^2_{1-\frac{\alpha}{2}, N-1}}}, \bar{\sigma} \cdot \sqrt{\frac{N-1}{X^2_{\frac{\alpha}{2}, N-1}}} \right]$$

In this

$$P \left\{ X^2_{\frac{\alpha}{2}, N-1} \leq \frac{(N-1) \bar{\sigma}^2}{\sigma^2} \leq X^2_{1-\frac{\alpha}{2}, N-1} \right\} = 1 - \alpha$$

As a result, if random variable obey normal distributions, when mean sample values, variance reliabilities, and confidence zones are given, it is then possible to know minimum values for sample numbers needed.

For the sake of convenience, this article gives mean values

respectively associated with degrees of reliability of 80%, 90%, 95%, and 99%. Variance reliability zone upper and lower limit graphs are, respectively, as shown in Fig.2 and Fig.3. In these, N is sample number, ψ is degree of confidence $1-\alpha$; $\delta_m = \frac{\bar{m}}{\sigma}$. The unit is $\frac{\bar{m}}{\sigma}$.

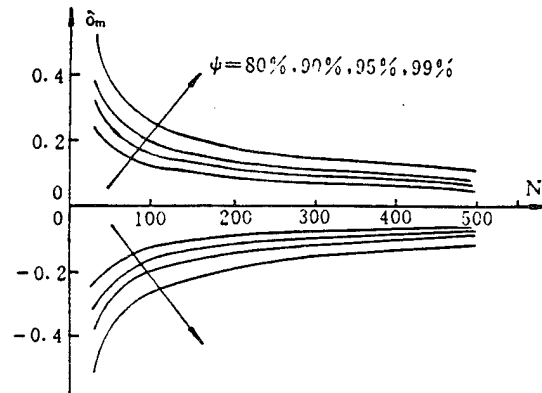


Fig.2 Mean Value Confidence Zone Upper and Lower Limit Graphs

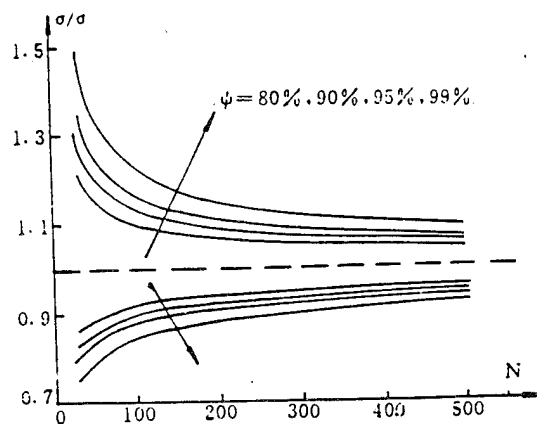


Fig.3 Variance Confidence Zone Upper and Lower Limit Graphs

(2) Normal Distribution Checks When carrying out normal distribution checks on random variables, "x2 goodness of fit checks" are one type of method which can be adopted. For detailed content, see reference [1]. In general, when doing research on guidance precision, random variables studied can, in all cases, be believed to follow normal distributions.

(3) Production of Random Numbers Random numbers are used in order to produce random initial state sets. With regard to the production of normal random numbers, there are many types of methods. As far as this article is concerned, after producing 0 ~ 1 range uniform random numbers by "multiplicative congruence methods", through transformations arrived at, the model can be displayed simply as:

$$\begin{aligned}
 \text{SEED} &= \text{MOD}(16807 \times \text{SEED}, 2147483647) \\
 X &= \text{SEED} \times 0.4656612875 \times 10^{-9} \\
 \text{SEED} &= \text{MOD}(16807 \times \text{SEED}, 2147483647) \\
 Y &= \text{SEED} \times 0.4656612875 \times 10^{-9} \\
 U &= \sqrt{-2 \ln X} & V &= 2\pi Y \\
 \text{RAND1} &= U \cdot \cos V \cdot P + \bar{X} & \text{RAND2} &= U \cdot \sin V \cdot P + \bar{X}
 \end{aligned}$$

In equations, before calculations, SEED acts as initial seed location value. The initial values can be any arbitrary whole number. P and \bar{X} are, respectively, variances and mean values associated with random numbers.

3. Introduction of Error

This article primarily studies the influences of guidance precision, target measurement precision, and terminal guidance control capabilities on systems. The errors are primarily guidance errors and target measurement errors. Due to guidance method errors embodying themselves in system models, we will, therefore, not discuss them a great deal here. Guidance

equipment errors and specific designs are related. What is discussed here are equipment errors associated with quick connection inertial guidance systems composed of laser gyroscopes (other plans can be analyzed in a similar way). After carrying out error compensation in inertial measurement combinations, equipment errors can be simply displayed by the use of the model below

$$\begin{array}{lll} \overline{W}_x = E1 + E2 \cdot W_x & \overline{\omega}_x = E7 + E8 \cdot \omega_x & \overline{\Phi} = \Phi + E13 \\ \overline{W}_y = E3 + E4 \cdot W_y & \overline{\omega}_y = E9 + E10 \cdot \omega_y & \overline{\Psi} = \Psi + E14 \\ \overline{W}_z = E5 + E6 \cdot W_z & \overline{\omega}_z = E11 + E12 \cdot \omega_z & \overline{\lambda} = \lambda + E15 \end{array}$$

In equations, E1, E3, E5, E2, E4, and E6 are, respectively, zero bias and calibration factor errors associated with three /56 acceleraometers. E7, E9, E11, E8, E10, and E12 are, respectively, drift and calibration factor errors associated with three gyroscopes. W_x , W_y , and W_z are actual apparent accelerations. \overline{W}_x , \overline{W}_y , \overline{W}_z are calculated apparent accelerations. ω_x , ω_y , ω_z are actual angular velocities.

$\overline{\omega}_x$, $\overline{\omega}_y$, $\overline{\omega}_z$ are calculated angular velocities. E13, E14, and E15 are initial alignment errors. Φ and Ψ are actual attitude angles. $\overline{\Phi}$, $\overline{\Psi}$ are calculated attitude angles. λ is azimuth alignment angles. $\overline{\lambda}$ is actual alignment angles.

\overline{W}_x , \overline{W}_y , \overline{W}_z and $\overline{\omega}_x$, $\overline{\omega}_y$, $\overline{\omega}_z$ directly substitute into ballistic calculation models. $\overline{\Phi}$ and $\overline{\Psi}$ are introduced in integral initial values. $\overline{\lambda}$ --going through an angle of rotation which is a coordinate transformation associated with E15--is added into system models.

Target track measurement errors can be taken to be ones that obey oblate spherical normal distributions. Prolate spheroids are target velocity vector directions. Assuming that the X axis represents target velocity vector direction, the Y axis is perpendicular to the X axis in the plane of target flight. The Z axis is determined from X and Y in accordance with right hand

rules. Then, calculated target position is:

$$X = E16 \qquad Y = E17 \qquad Z = E18$$

Through coordinate transformations, (X,Y,Z) are taken and transferred into target flight coordinate systems. Together with target measurement values, they are substituted into system models.

4. Statistical Set Selection

Corresponding to a random initial state set--through system simulation models--it is possible to simulate and calculate a set of flight trajectories. If one uses certain types of conditions to act as restraints, then, various trajectories fitting with each parameter associated with these constraining conditions then form each set. Statistical characteristics associated with this set show the precision of this parameter. For example, adopting the instant $t = T$ to act as constraining condition, X coordinates act as the studied parameters. Then, X coordinates associated with various trajectories at instant T will comprise a set. The mean square deviation of this set then shows the distribution precision of various trajectories associated with the X direction at instant T . As far as constraining conditions which are not the same are concerned, sample spaces for the same parameter are different. Dispersed statistical results are also not the same. As a result, the selection of constraining conditions directly influences statistical results. With regard to talking in terms of antiballistic type weapons system precisions, if one uses standard trajectories (trajectories when there are not any errors) at a certain instant T to act as constraining conditions, when the studied parameters are X, Y, Z values associated with instant T , research results are excessively conservative. The reason is that, dispersion associated with missile velocity vector direction has very small influences on target hit precisions, but velocity vector normal scattering has very large influences on target hit precisions. Taking T to be the

constraining condition, as far as obtained sample spaces are concerned, there is no way to consider this factor. The constraining conditions selected in this article are--when there is no terminal guidance--missile-target relative velocities and relative distances perpendicular to each other. Studied parameters are missile-target relative vectors. Speaking in theoretical terms, these are position errors which terminal guidance must correct. Due to position vectors possessing directional characteristics, it is, therefore, necessary to carry out projection of distance vectors onto inertial coordinate systems, obtaining distance vector values in the three axis directions. The values in these three axial directions respectively construct three sample spaces (From normal distribution checks, one learns that the three sample spaces follow normal distributions.) In conjunction with this, doing three dimensional combined normal distribution analysis, it is possible to obtain covariance distance array D.

If one makes a conservative estimate, one then selects mean square variations to be:

$$\sigma = \sqrt{D(1,1) + D(2,2) + D(3,3)}$$

If, during precise calculations, one then solves for the long and short axes of combined oblate spheroid normal distributions--in accordance with definite probabilities--calculations are made of spheroid dispersion magnitudes.

Table 1 is statistical results (data done with unitary treatments) associated with studied parameters which are taken to be missile-target relative distance as well as locations X,Y,Z, using standard trajectory target impact instants to be constraining conditions (constraining condition 1) as well as taking missile-target relative velocity vectors and relative distance vectors being perpendicular to each other to be a constraining condition (constraining condition 2).

Table 1 Contrasting Table of Statistical Results Under Different Constraining Conditions

	(3) 相 对 距 离				X	Y	Z	总 (4) 和
	X方向 (2)	Y方向 (2)	Z方向 (2)	总 (4) 和				
① 条件 1	1.053	0.803	0.610	1.458	0.874	0.805	0.600	1.331
① 条件 2	0.093	0.813	0.576	1.000	0.721	0.837	0.600	1.256

Key: (1) Condition (2) Direction (3) Relative Distance
(4) Grand Total

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From the table, it can be seen that statistical results obtained under different constraining conditions are very greatly different from each other.

With regard to the requirements of different precision analyses, it is possible to select corresponding constraining conditions. If a target distance of S kilometers is the restraining condition, and the included spacial angle (line of sight angle) between missile axes and missile target relative distance vectors is the studied parameter, it is possible to analyze whether or not terminal guidance target acquisition sight field sizes satisfy requirements. On the basis of the requirements of different precision analyses, constraining conditions and research parameters--in actual applications--can be flexibly selected.

III. METHOD APPLICATIONS

Making use of Montecarlo methods, it is possible to study many types of precision problems. It has already been pointed

out above that, through selecting different statistical sets, that is, selecting different constraining conditions and research parameters, it is possible to reach different objectives. This article uses a certain type of ballistic model as its foundation, studying two problems in the area of precision. In all cases, data in these has unitary treatments carried out on it.

1. Uses in Solving for Terminal Guidance Capabilities

Initial state sets are produced on the basis of a certain type of given characteristic indices. Targets are low orbit satellites at given altitudes. There are two constraining conditions, one is that relative missile-target velocities and relative distances are perpendicular to each other. The studied parameter is missile-target relative distance \vec{R} . The other is a given kilometer distance from missile to target. The research parameter is the included spacial angle θ between missile axes and missile-target relative distance vectors. Statistical reliability is taken as 90%. The reliability zone is selected as $|\bar{m}-m| < 0.1 \bar{\sigma}, 0.9 < \bar{\sigma}/\sigma < 1.1$. The sample number obtained is selected as at least 300. Making use of Montecarlo methods--after going through ballistic simulation calculations--statistical analysis gives mean square deviations as:

$$\begin{aligned}\sigma_R &= 1 \\ \sigma_\theta &= 0.1\end{aligned}$$

The factor of assurance is selected to be 1.2, and 3σ is used to do calculations. By contrast, $\theta_{\max} = 0.36$ satisfies system requirements. $R_{\max} = 3.6$. Due to the fact that distances associated with terminal guidance effects are given, missile-target relative velocities for this time can be calculated from trajectories. As a result, periods associated with terminal guidance effects are known. In accordance with guidance methods associated with terminal guidance, on the basis of $R_{\max} = 3.6$, it is then possible to calculate estimates for the magnitude of

control forces associated with terminal guidance, thereby obtaining capabilities needed for terminal guidance.

2. Uses in Studying Match Ups Between Various Performance Indices Related to Precision

As far as research on matching between various performance indices associated with precision is concerned, in actuality, it is nothing else than the study of sensitivities associated with various characteristic indices. Sensitivities are the influences of single performance indices on precision when other performance indices do not change. Through studies of sensitivities, it is possible to cause the rationalization of the distribution of various performance indices in order to reach maximum cost effectiveness ratios. Here, analyses are done when target altitudes are given for sensitivities associated with track measurement error T , accelerometer bias error $A0$, accelerometer calibration factor error $A1$, gyroscope zero drift $D0$, and calibration factor error $D1$. As far as precisions are concerned, they are represented by missile-target relative distance R . Such factors as constraining conditions, sample numbers, and so on, are the same as before. Due to limitations on the scope of this article, calculation results are only given as sensitivity graphs for accelerometer zero bias errors $A0$ and gyroscope zero drift $D0$, as shown in Fig.4 and Fig.5.

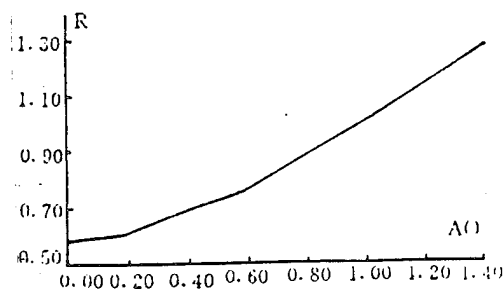


Fig.4 Sensitivity Graph for Accelerometer Zero Bias Error $A0$

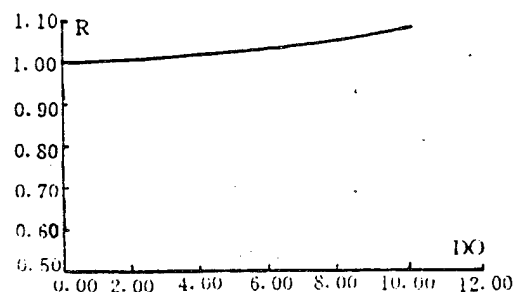


Fig.5 Sensitivity Graph for Gyroscope Zero Drift DO

From the graphs, it can be seen that DO sensitivities are small. AO sensitivities are large. Taking DO and relaxing it to 10--after calculations--it is known that ΔR only increases 5%. Generally speaking, when indices for various sensitivities are appropriate, index matches are relatively good. In actual situations, it is also necessary--in accordance with expenses needed to improve or relax certain indices or the size of profits obtained--to make comprehensive considerations and finally rationally present various system performance indices.

IV. CONCLUSIONS

From this, it can be seen that--utilizing Montecarlo methods--it is possible to effectively, simply, and accurately study various types of precision problems. Today--with the developments in high speed computer calculations--they will certainly achieve widespread utilization more and more.

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